Operations Planning and Formation Flying: Analyzing Resource Usage in Formation Assembly

Brian Engberg
Stanford University
Robert Twiggs, Advisor

Abstract

Increasing interest in small spacecraft has fostered a variety of new mission concepts. Within this busy framework, one particular family of mission concepts is taking shape – spacecraft formations. Formation flying entails the organized collection of satellites in a region of space to do a certain job. Missions of this kind pose some interesting design challenges. Most notably, active control of a formation is a fairly popular topic. The aim of such research is to devise a means of stabilizing a loosely bound system while conserving valuable resources. Another area, which seems to be lagging in attention, is the area of formation assembly. Formation assembly involves getting the vehicles to some rendezvous point prior to actually constructing the formation. Equal care should be taken with system resources at this stage. In this paper, a brief overview of these formation flying concepts will be given. The discussion will then focus on analyzing a hypothesized formation assembly strategy. The emphasis will be on using precious resources such as time and fuel as a performance metric for the strategy under various conditions. The derived relations will provide insight about what one might expect to achieve operationally when attempting to assemble a group of satellites.

Introduction

Aerospace engineers endeavor to use new technologies in order to develop more intricate space missions. When applied, these missions help to meet the ever-increasing complex goals of advancing technology. One example of this type of symbiosis is spacecraft constellations, or large groups of similar spacecraft. Recent advances in technology have reduced the size and lowered the power consumption of electronic components and sensors. This has made it feasible to mass-produce and launch fleets of small space vehicles. Size is an issue, since a smaller satellite will generally cost less, require fewer resources to build and operate, and can be more easily mass-produced than a single, large satellite. Constellations can also have some performance benefits over their larger, singular counterparts, especially when it comes to issues such as area of coverage and reliability. When launched, fleets of these small vehicles can be used to meet other technological demands of society, such as the increased demand for sharing information and data around the planet.

Within the scientific community, there is great interest in a particular subset of spacecraft constellation missions. That subset is the one of spacecraft formations. Spacecraft formations are associations of space vehicles that arrange themselves so as to perform a particular function. Each individual vehicle is an important part of a larger system. Individual vehicles may operate independent of their counterparts, but all vehicles must operate and communicate together to achieve the overall goals. When spacecraft are arranged in a particular formation as part of their intended mission, they are said to be participating in "formation flying." Formation flying has a wide range of useful applications. Examples include stereoscopic measurement of the Earth's atmosphere and its surface, interferometric imaging of deep-space objects, and gathering real-time data of field quantities, such as the Earth's magnetic field.

But, challenges do exist. First and foremost is the challenge of knowing and controlling the absolute and relative positions of the spacecraft within the formation. Certainly the task of precisely determining a spacecraft's location is a formidable one, but this problem is then compounded by the need to communicate that position to all other spacecraft in the group (and in turn interpret their positions) and then act appropriately on that information. Solving for the absolute and relative attitudes of the spacecraft poses similar problems. Another important topic, and one that will be a focus of this discourse, is that of resource management.

Resource management is essentially the attempt to know (and if possible, control) the consumption and distribution of mission-critical quantities. Examples of these quantities include power, fuel, mission time, and CPU memory. It is important to have such information available, since a failure or lack of resources on one vehicle may prevent a formation from achieving its overall mission. This is especially the case when considering a resource like fuel, which cannot be replenished. In order to prepare, the design team must carefully analyze its estimated resource constraints, and then choose appropriate operations plans or algorithms. For example, hypothesize two control algorithms under a specific flight condition. In algorithm 1, spacecraft A uses 4 units of fuel while spacecraft B uses 1 unit. In algorithm 2, both spacecraft use 3 units. Algorithm 1 is attractive because it uses less overall system fuel (5 units vs. 6 units), but algorithm 2 distributes fuel usage evenly, which may be more desirable if the given flight condition is to be encountered multiple times. Otherwise, spacecraft A will run out of fuel before B (unless it is designed to carry more fuel, but that's a different topic).

The Space Systems Development Laboratory (SSDL) at Stanford University provides an excellent atmosphere in which to encounter and analyze these types of problems. The main aim of SSDL is to provide students with a broad range of space system engineering skills. This includes the design and construction of small, innovative spacecraft. Researches are also conducted in the fields of spacecraft operations and space systems simulations. Beginning students in the lab are given the opportunity to work as part of an all-student design team on a Satellite Quick-Research Testbed, or SQUIRT satellite. These satellites integrate commercial off-the-shelf type components into a platform capable of demonstrating new and innovative spacecraft technologies. Advanced students are then able to work on more complex, detailed satellite projects, such as the Orion program.

The Orion mission is a low-cost mission aimed at developing a system of three satellites [Ref 2]. The trio of space vehicles will be capable of performing a formation flying mission. The project team will identify and implement technology available to achieve this goal. A low-cost, low-power, multi-channel GPS receiver will be used to determine each satellite's absolute position and attitude. The exchange of position and attitude data will be handled by a special cross-link communication system. Pre-planned, organized maneuvers and formation control algorithms will be commanded at a high-level from the ground, and will be governed locally by real-time, autonomous flight control software. Formation adjustments and control thrusts will be performed by a cold-gas jet propulsion system. The Orion system will demonstrate repeatability by assembling and disbanding the prescribed formation a number of times over the projected one- to two-month mission lifetime. The Orion mission objectives and resource constraints have provided some of the motivation behind the investigations presented here.

Research Focus

To summarize, the interest in using an economy of scale in space missions has been cited. The interpretation is that it is desirable to mass-produce and launch large quantities of smaller spacecraft to accomplish the same job as a single, large spacecraft. In this manner it is possible to lower costs while increasing other desirable system characteristics such as robustness and area of coverage. Furthermore, it has been noted that organized, formation flying clusters of small spacecraft would provide a useful tool for scientific exploration and military surveillance. Individual vehicles would form the disconnected parts of a larger, "virtual" spacecraft. A communications scheme, rather than a data bus, provides the connection between parts. The Stanford Orion project aims to verify these assertions by successfully flying a set of small space vehicles in a controlled formation.

Once the motivation behind constellations of small satellites has been suitably justified, attention may be turned towards some of the more detailed issues. Certainly one of the most obvious challenges has to be that of actively controlling the spacecraft. The effect of the space environment on these distributed systems will be a new avenue to explore. As will be shown later, small variations in orbit size between spacecraft will correspond to slight variations in orbital period. Minute differences in atmospheric drag and gravitational forces encountered by individual spacecraft can build up over time. A single, large spacecraft would account for these perturbations as part of its dynamic behavior. A system of smaller vehicles has no physical connections, and therefore exhibits a much less desirable behavior: it flies apart! In addition, determining the position of all satellites appears to be a difficult process at best. However, as suggested earlier, technology advances offer a great deal of aid. The Global Positioning System (GPS), a constellation of 30 earth-orbiting satellites, can be used to determine absolute position to near-meter-level accuracy. Application of this technology to satellite formations in low-Earth orbit (LEO) can provide the sensing precision and accuracy needed. Moreover, low-power flight computers with high-capacity throughput are becoming available. This affords each spacecraft the processing ability required to run complex control algorithms. Both of these solutions are part of the baseline design for the Orion mission.

A fair amount of work is being conducted in the field of formation flying and control. On the other hand, an equally important area that seems to elude attention is formation assembly. Formation assembly is intended to describe the process of collecting the constellation of satellites to a given region of space just prior to constructing any formation. For the Orion mission, collecting the three satellites together prior to executing the formation flying experiment is expected to consume a fairly large portion (up to 20%) of the system fuel. Or, as an example, consider a collection of satellites deployed from a launch vehicle. These satellites may not immediately assemble into a formation, as the ground operators may desire a "check-out" phase or other period of time which will give the system a chance to slowly drift apart. Or, consider the fact that a collection of satellites may be deployed from TWO launch vehicles. Surely there can easily be a significant difference in the deployment orbits that will require a suitable assembly strategy. Also, consider then the cost of maintaining a formation. For the Orion mission, maintaining a simple formation for less than an orbit is estimated to consume up to 15% of the system fuel. Presumably, a more advanced formation flying mission will require numerous scientific measurements over a much longer period of time. As a result, the formation must be maintained for that period (and so consuming large amounts of resources), or else, after each experimental measurement, the formation control is broken and the system is allowed to drift or enter some other type of "cruise" configuration. Since a cruise configuration would seem to consume fewer resources, an efficient re-assembly algorithm would help to increase the mission lifetime. In this respect it is clear that studying effective and efficient ways to rendezvous satellite systems will be very useful.

For example, if two satellites need to rendezvous, one solution is to perform an enormous fuel burn on one satellite. The vehicle quickly zips over to the other satellite, and then stops. This method is very effective (short time to complete) but horrendously inefficient (large fuel consumption). On the other hand, the first satellite can expend no fuel and continue to drift away until someday it has drifted a complete lap around the Earth, meets up with the second satellite, and with a minute puff of fuel, completes its rendezvous maneuver. This method is extremely efficient from a fuel standpoint. However, it is not very effective, since the maneuver takes long enough for pigs to actually evolve wings. In any case, formation assembly can be boiled down to a case of ensuring that critical system resources are expended cautiously. These system resources may include fuel, power, memory, and mission time.

In this study, the focus will be on developing analysis tools for fuel consumption and rendezvous time for a particular formation assembly strategy. Drawing upon the earlier discussion, it will be assumed that assembly will take place in a LEO environment, and that all participating spacecraft are, more or less, identical. The produced results will attempt to show the relationships between the consumption of fuel and time and the choice of the assembly point for the given algorithm. Results can be applied to an Orion-like mission to see if the estimated fuel requirements and mission time are satisfied. But first, there is need of some mathematical tools to accomplish this goal.

Analysis

Orbital mechanics is a widely studied subject. It will be assumed by and large that the reader is familiar with some of the basic concepts associated with the field. However, a brief description of some important parameters is in order, followed by a review of some important orbital equations that will serve as useful analysis tools.

Consider a satellite in orbit around some body, such as the Earth. If the origin of a three-axis inertial frame is placed at the center of the Earth, the position and velocity of the satellite relative to that origin can be represented. The three-dimensional vectors of position and velocity completely determine the satellite's orbit. The magnitudes of these vectors determine the orbital energy, and the cross product of the vectors set the angular momentum. The six quantities contained in these two vectors are not the ones most often used to describe an orbit, however. Instead, six other quantities, known as the Keplerian elements, are employed. The Keplerian elements are a set of six numbers that describe the size, shape, and orientation of an orbit in space relative to the central body. They can be derived from a given initial position and velocity. To summarize, the elements are (see Figure 1):

- Semi-major axis (**a**) – All closed orbits are ellipses. The value of **a** is equal to half the length of the ellipse's long axis. Thus, **a** determines the size of an orbit
- Eccentricity (**e**) – This is a measurement of how elongated the orbital ellipse will be. Eccentricity therefore determines the shape of an orbit. A zero value of **e** means the orbit is circular.
- True anomaly (**Θ**) – This parameter measures the angle in the plane of the orbit between the radial vector of the satellite's location, and that same vector when the satellite is at its closest approach to the orbiting body (known as the periapse).
- Inclination (**i**), Argument of periapse (**ω**), and Right ascension of the ascending node (**Ω**) – these three angles determine the orientation of the orbit in three-dimensional space.

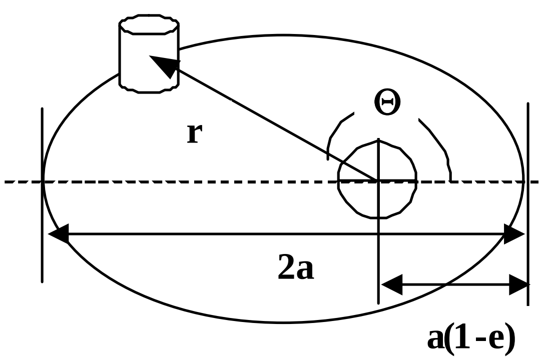

Figure 1 – An example orbit

Figure 1 illustrates the first three parameters mentioned. The focus of the analysis will be on planar orbits, without regard to orientation. Thus, only the first three parameters are relevant to the discussion, especially **a**.

The first tool to be used was provided by a German astronomer, Johannes Kepler, in the early 17th century. Note that the orbital parameters mentioned above bear his name. Kepler's main contribution was a set of three "empirical" laws that described the motions of the planets. Note that these laws are not truly empirical, as each was found later (by Newton) to have a basis in physics. Kepler's third law states that "the squares of the orbital periods of the planets are proportional to the cubes of their mean distances to the sun." [Ref 1]. As mentioned, this law was later derived from physical principles, and shown to be true for all orbited bodies (not just the sun). The mathematical representation of this law is:

$$t^2 = \frac{4\pi^2}{\mu} a^3 \qquad (1)$$

Here, **t** is the orbital period, **a** is the semi-major axis length, and **μ** is a gravitational constant. The value of **μ** is different for each body orbited (i.e. Earth, Mars, the Sun). The value of **μ** for the Earth is 3.986 x 10^5 km^3/sec^2. A derivation of this relation can be found in most books on orbital mechanics, such as reference x.

The second tool to be used is a well-known relation in the field of orbital mechanics. It is known as the "vis-viva" equation. The vis-viva equation is derived from the fact that the total mechanical energy in a closed orbit is conserved. This equation relates the magnitude of an object's velocity to its radial distance from the orbited body, and can be written as:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \qquad (2)$$

Here, **v** is velocity and **r** is the separation between the centers of mass of the satellite and the orbited body. **μ** and **a** are the gravitational constant and semi-major axis of the orbit, as explained previously.

Of particular interest is the specific mechanical energy of an orbit, or the sum of the kinetic energy and (negative) gravitational potential energy per unit mass. The vis-viva equation is particularly useful in calculating this conserved quantity:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = \mu\left(\frac{1}{r} - \frac{1}{2a}\right) - \frac{\mu}{r}$$

or $$\varepsilon = \frac{-\mu}{2a} \qquad (3)$$

The specific energy of an orbit is given by equation (3). It is evident that orbital energy is determined wholly by the size of an orbit (i.e. the parameter **a**), and vice-versa. So, if a thrust or perturbation force added or subtracted from the total energy of an orbit, **a** would change accordingly. From equation (1) it is evident that there will be a corresponding change in the orbital period. Recalling that differences in orbital period are the source of permanent drift in formation flying, it is clear that a significant change in orbital size or energy is to be avoided. So what produces such change?

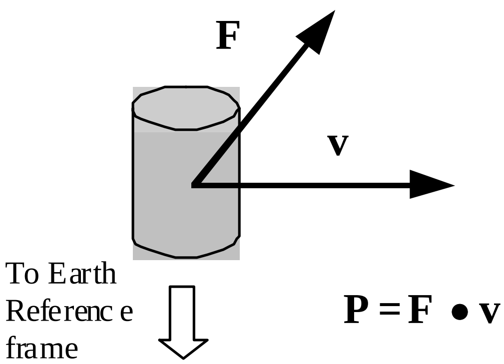

Figure 2 – Illustration of Force-Energy relationship

Consider again an inertial frame of reference centered at the Earth's core. A space vehicle is traveling with a velocity vector **v** relative to this frame of reference, and experiences an external force **F**. This force may be an environmental or other perturbing force, or it may be due to a thrust generated internally by the vehicle. Simple physics dictates that the power, or the rate at which energy is added to the space vehicle, is given by the dot product of **F** and **v**. Clearly, if **F** and **v** are perpendicular, then no energy is added to the orbiting body. Thus, no change in orbit size or period will occur. However, it is clear that in-track force components (components of force acting in the direction of travel) will cause a change in orbit energy, and therefore, the period. So, if the spacecraft is tracking a particular reference point, an in-track force component will cause the spacecraft to drift away from that point. This indicates that only thrusts and disturbances that occur along the in-track direction will contribute to permanent drift. All other contributions (forces in the out-of-plane and radial directions) will produce drifts that are cyclical. For this reason, the following analysis will ignore dynamics occurring out of the orbital plane. The discussion will focus on in-plane dynamics and maneuvers.

The problem at hand will therefore be set up as shown in Figure 3. The target assembly point C moves along a circular reference orbit of radius R as measured from the center of the Earth. Since the discussion is limited to in-plane orbit changes, there are four positions, relative to C, that a space vehicle may have: (I) a higher orbit and lagging behind C, (II) a higher orbit and leading C, (III) a lower orbit and leading C, and (IV) a lower orbit and lagging C. The goal for each condition is the same. This goal is to maneuver through space in such a way as to match the position and velocity of C. Whatever method used to achieve this goal can be applied to individual spacecraft, as this is a single-vehicle problem. The formation assembly problem, taking all vehicles into account, then becomes one of choosing an appropriate assembly point (orbit).

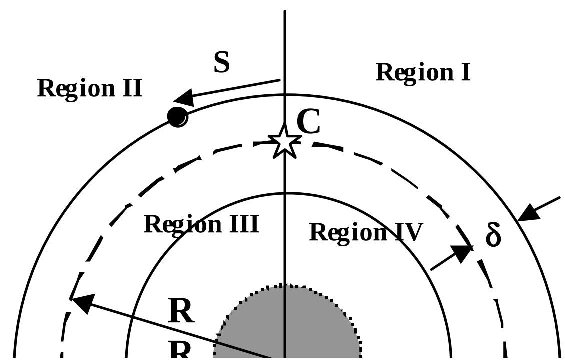

Figure 3 – The formation assembly problem

Before solving this problem, the assembly strategy must be defined more clearly. Since assembly is assumed to take place in a LEO environment, it will also be assumed that the spacecraft orbits, as well as the target orbit, are circular. This is a fairly reasonable approximation for LEO orbits, and will serve to make calculations a bit easier. In addition, it will be assumed that all orbit adjustments will be relatively small ones. The endeavor here is not to analyze the problem of gathering vehicles separated by half a cosmos. The previous arguments made about orbit injection and re-assembly after cruise will be honored, which means that the concern will be with slight orbit variations. As will be seen later, this assumption also allows the most of the working equations to be linearized. Finally, in order to keep pace with the desire to efficiently use resources, it will be assumed that all orbit transfers will be Hohmann transfers. A Hohmann transfer (H-transfer) is a two-impulse-burn orbit transfer that uses the minimum amount of fuel. [see Ref 1,3] This is true for transfers between circular orbits, which has been assumed. In an H-transfer, the satellite completes exactly one half of an elliptic orbit that intersects both the original and target orbit.

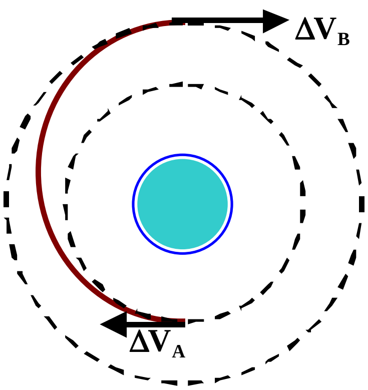

Figure 4 – A Hohmann transfer

Now comes the task of solving the problem at hand. First, consider region II (refer to Figure 3). The space vehicle resides in a circular orbit above the reference orbit by a small amount δ (Let $\delta > 0$ indicate that the orbit is higher than the reference orbit). Also, the vehicle leads the assembly point by a small amount S (Let $S > 0$ indicate a satellite lead in the direction of orbit). Clearly, the reference point is catching up to the spacecraft, and so the fuel-optimal strategy will be to wait until the reference point is in such a position that a single H-transfer can be made down to the assembly point. The following method shows how to calculate the desired performance metrics:

First, consider the lead angle of the spacecraft. Since S is part of a circular arc, the angle subtended is $\frac{S}{R+\delta} \approx \frac{S}{R}$. Let θ be the measured from the initial assembly point radial line to the space vehicle radial line. Let θ_C be measured likewise for the assembly point itself. Define α as the critical lead angle. When the lead angle is α, an H-transfer to the target orbit will result in a rendezvous between the satellite and the assembly point.

Thus, at time t = 0,

$$\theta(0) = \alpha \qquad \text{and} \qquad \theta_C(0) = 0$$

After the H-transfer (elapsed time t_H),

$$\theta(t_H) = \alpha + \pi \qquad \text{and}$$

$$\theta_C(t_H) = \dot{\theta}_C t_H$$

where $\dot{\theta}_C$ is the angular rate of the circular reference orbit. The goal is to match these two angles, whereby the value of α may be determined:

$$\theta = \theta_C \Rightarrow \alpha = \dot{\theta}_C t_H - \pi \qquad (4)$$

For a circular orbit, **a** = R = constant. It is also known that for an object traveling along a circular path, the speed is equal to the angular rate multiplied by R. Using this knowledge and equation (2) gives:

$$v = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{R}\right)} = R\dot{\theta}_C$$

$$\Rightarrow \dot{\theta}_C = \sqrt{\frac{\mu}{R^3}} \qquad (5)$$

The H-transfer time is simply one-half of an orbit whose semi-major axis is the average of the original and target orbits:

$$t_H = \frac{1}{2}\sqrt{\frac{4\pi^2}{\mu}}a^{3/2} = \pi\sqrt{\frac{(R+\delta/2)^3}{\mu}}$$

$$= \pi\sqrt{\frac{R^3}{\mu}}\left(1 + \frac{\delta}{2R}\right)^{3/2}$$

Invoking the assumption of small orbit changes, it is clear that δ is very small compared to R. As such, this equation can be made linear in δ by approximating the expression, using the first two terms of a Taylor series expansion:

$$\frac{\delta}{R} << 1 \Rightarrow t_H \approx \pi\sqrt{\frac{R^3}{\mu}}\left(1 + \frac{3\delta}{4R}\right) \qquad (6)$$

Combining equation (4), (5), and (6) gives the value of α:

$$\alpha = \pi\left(1 + \frac{3\delta}{4R}\right) - \pi = \frac{3\pi}{4}\frac{\delta}{R} \qquad (7)$$

This value of α defines the point at which the H-transfer will take place. Recall that in this region, the assembly point is overtaking the satellite (S is decreasing). The mandate of the chosen algorithm is to wait until the separation angle reaches this mark and then conduct the appropriate transfer. But what does this mean for the chosen performance metrics? The relationships between δ, S, fuel consumption and rendezvous time must be derived.

The total rendezvous time for a transfer in this region is equivalent to the H-transfer time plus the time required for the system to attain the appropriate configuration. The initial angular separation (determined by S) less α is the angle through which the assembly point must "catch up" for this configuration to occur. The rate at which this angular separation is reduced is determined by the difference in angular rates between the two orbits. Mathematically:

$$t_{RDVZ} = t_H + \frac{\frac{S}{R} - \alpha}{\dot{\theta}_C - \dot{\theta}} \qquad (8)$$

Using the small orbit change approximation, it can be shown that:

$$\begin{aligned}\dot{\theta} &= \sqrt{\frac{\mu}{(R+\delta)^3}} = \sqrt{\frac{\mu}{R^3}}\left(1+\frac{\delta}{R}\right)^{-3/2} \\ &= \sqrt{\frac{\mu}{R^3}}\left(1-\frac{3\delta}{2R}\right)\end{aligned} \qquad (9)$$

Subtracting this result from equation (5) and plugging in to equation (8) gives:

$$\begin{aligned}t_{RDVZ} &= t_H + \frac{S - 3\pi\delta/4}{R}\sqrt{\frac{R^3}{\mu}}\,\frac{2R}{3\delta} \\ &= \sqrt{\frac{R^3}{\mu}}\left(\frac{\pi}{2} + \frac{3\pi}{4}\frac{\delta}{R} + \frac{2S}{3\delta}\right)\end{aligned} \qquad (10)$$

This is the solution for total rendezvous time under the conditions δ > 0 and S > 3πδ/4. Take special notice of the latter constraint. If this condition is violated, the second term in equation (10) will produce a negative time contribution. This is a physical impossibility. The interpretation of this constraint is that there exists, for each δ, a positive separation lead S for which no H-transfer solution is possible. In other words, the initial separation angle is already less than α.

The rendezvous time is only the first performance metric. The metric for fuel usage will now be solved. When executing orbit maneuvers, some force is applied to the spacecraft so as to change its velocity. The term for this change is "delta-V", or Δv. Since fuel is expended in this effort, delta-V is a way to represent the amount of fuel needed by a spacecraft to perform a given action. Propulsion systems are often described by the total amount of delta-V they can supply.

For the region II problem, two impulsive burns must be evaluated and added. Equation (2) will be very useful in this regard. For the first impulsive burn, the satellite is traveling in a circular orbit, and will slow down by a small amount so that it is on the Hohmann transfer ellipse. The change in velocity is:

$$\Delta v_1 = \sqrt{\frac{\mu}{R+\delta}} - \sqrt{\mu\left(\frac{2}{R+\delta} - \frac{1}{R+\delta/2}\right)} \qquad (11)$$

The first term is the spacecraft's speed just before the burn, and the second term is the spacecraft's speed at the same point just after the burn. Note that the appropriate value for the transfer ellipse's semi-major axis has been provided. It is usually a stated assumption that impulsive transfers are modeled as "instantaneous" changes in velocity. This is not true, but it is a good approximation as long as the burn time is short compared to the transfer time.

The second impulsive burn can be analyzed in the same manner:

$$\Delta v_2 = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{R+\delta/2}\right)} - \sqrt{\frac{\mu}{R}} \qquad (12)$$

The total change is then:

$$\begin{aligned}\Delta v &= \Delta v_1 + \Delta v_2 \\ &= \sqrt{\frac{\mu}{R}}\Big\{(1+\frac{\delta}{R})^{-1/2} - (1+\frac{\delta}{R})^{-1/2}(1+\frac{\delta}{2R})^{-1/2} \\ &\quad + (1+\frac{\delta}{R})^{1/2}(1+\frac{\delta}{2R})^{-1/2} - 1\Big\}\end{aligned}$$

This can be linearized to:

$$\Delta v = \sqrt{\frac{\mu}{R}}\left\{(1-\frac{\delta}{2R}) - (1-\frac{3\delta}{4R}) + (1+\frac{\delta}{4R}) - 1\right\}$$
$$= \sqrt{\frac{\mu}{R^3}}\frac{\delta}{2} \qquad (13)$$

This is the solution for the fuel consumption metric for the conditions δ>0 and S>3πδ/4. Note that while the rendezvous time depends on the initial separation, the fuel expenditure does not. This is due to the fact that S is immaterial to the actual orbit transfer; the H-transfer will be executed at a point determined solely by δ. On the other hand, the length of time it takes to get to that point is determined by S.

Now let us consider region IV (see Figure 3). In this region, a behavior similar to region II occurs, except this time, it is the spacecraft that is overtaking the rendezvous point. Rigor might demand that a discussion similar to the previous one be presented, but some quick arguments can save time and effort.

In region IV, S, δ < 0. What does this change form the previous discussion? Since the spacecraft is overtaking the assembly point, there is obviously a point at which the spacecraft can simply H-transfer up to the target. However, there will now be some sign changes. The initial conditions become:

$$\theta(0) = -\alpha \qquad \text{and} \qquad \theta_C(0) = 0$$

After the H-transfer, the space vehicle will be at:

$$\theta(t_H) = \pi - \alpha$$

and so the solution to α will be:

$$\alpha = \pi - \dot{\theta}_C t_H \Rightarrow \alpha = \frac{-3\pi}{4}\frac{\delta}{R}$$

but since δ < 0, it is clear that this is the same solution as before. Similarly, the denominator in equation (8) will change sign, since the angular rate of the spacecraft is now larger than that of the assembly point. But, this is accompanied by a sign change in the numerator, since the initial lag angle is now given by:

$$\frac{3\pi\delta/4 - S}{R} \qquad (14)$$

instead of $\frac{S - 3\pi\delta/4}{R}$ (recall that S < 0 also)

Thus, the rendezvous time in this case is also given by:

$$t_{RDVZ} = \sqrt{\frac{R^3}{\mu}}\left(\frac{\pi}{2} + \frac{3\pi}{4}\frac{\delta}{R} + \frac{2S}{3\delta}\right) \qquad (10)$$

The one item that *does* change is the constraint condition. Expression (14) must be positive in order to ensure that the H-transfer point has not already been passed. As a result, the constraint on S now becomes S < 3πδ/4, with δ < 0.

It is clear that for a region IV maneuver, the orbital energy must increase. Inspection of equations (12) and (13) makes it equally clear that as a result, the expressions for the delta-V burns are still correct, but will change sign. The post-burn velocity now becomes the pre-burn velocity, and vice-versa. As a result, the total delta-V for a region IV maneuver is:

$$\Delta v = \sqrt{\frac{\mu}{R^3}}\left(-\frac{\delta}{2}\right) \qquad (15)$$

This is the same solution as equation (13), since δ < 0.

Now the remaining regions can be solved. Consider a spacecraft in region I (see Figure 3). In this circumstance, the vehicle is above the reference orbit and lags the assembly point (δ > 0 and S < 0). In this case, it is clear that S will continue to increase as time goes on, unless an attempt is made to catch up. In order to accomplish this, the vehicle must H-transfer to an orbit inside the orbit of the assembly point. This is the only way it can hope to catch. An H-transfer can then be made back out to the assembly point. The main question, though, is how much inside the target orbit should the first transfer be?

At this point it becomes necessary to modify the assembly strategy slightly. Whereas in regions II and IV, the general approach was to save fuel (i.e. coast to a point in orbit ready-made for an H-transfer), the approach for region I (and III) will be to save time. The first impulse burn will put the spacecraft on an H-transfer orbit that will carry it inside the orbit of the assembly point. Upon arrival, the spacecraft will be located precisely at a critical lag angle (as per region IV). A transfer could be made to a different inner orbit, but such an exchange might require the satellite to coast to the H-transfer point. With the proposed strategy, the first transfer takes the vehicle directly there. In addition, this method requires only three impulsive burns instead of four.

The first order of business is to discover the relationship between the inner and outer orbits. Start with the initial conditions:

$$\theta(0) = \frac{S}{R} \quad \text{and} \quad \theta_C(0) = 0 \tag{16}$$

After the first H-transfer, the angular positions are:

$$\theta(t_{H1}) = \pi + \frac{S}{R} \quad \text{and} \quad \theta_C(t_{H1}) = \dot{\theta}_C\, t_{H1} \tag{17}$$

The desire is for the difference between these two angles at the arrival time to be equal to the corresponding critical angle for the inner orbit:

$$\theta_C(t_{H1}) - \theta(t_{H1}) = \alpha = \frac{-3\pi}{4}\frac{\varepsilon}{R} \tag{18}$$

Here ε is the separation between the intermediate inner orbit and the final target orbit. It is analogous to δ in the region IV problem. Once again, the travel time is the semi-period of the transfer ellipse:

$$\begin{aligned} t_{H1} &= \pi\sqrt{\frac{(R+\varepsilon/2+\delta/2)^3}{\mu}} \\ &= \pi\sqrt{\frac{R^3}{\mu}}(1+\frac{\varepsilon}{2R}+\frac{\delta}{2R})^{3/2} \\ &= \pi\sqrt{\frac{R^3}{\mu}}(1+\frac{3\varepsilon}{4R}+\frac{3\delta}{4R}) \end{aligned} \tag{19}$$

Combining this relation and equations (5), (17), and (18) yield the equation

$$\pi(1+\frac{3\varepsilon}{4R}+\frac{3\delta}{4R}) - \pi - \frac{S}{R} = -\frac{3\pi\varepsilon}{4R} \tag{20}$$

which can be solved explicitly for ε:

$$\varepsilon = -\frac{\delta}{2} + \frac{2S}{3\pi} \tag{21}$$

Again, δ > 0 and S < 0 in this region. Armed with this relationship, it is now possible to determine the relationships for the performance metrics.

Since the activity in this region is simply to conduct two back-to-back H-transfers, the total rendezvous time will be the sum of the two transfer times. Both of these values are known and have already been linearized. They are equation (19) above, and equation (6) with ε substituted for δ.

$$\begin{aligned} t_{RDVZ} &= \pi\sqrt{\frac{R^3}{\mu}}(1+\frac{3\varepsilon}{4R}+\frac{3\delta}{4R}+1+\frac{3\varepsilon}{4R}) \\ &= 2\pi\sqrt{\frac{R^3}{\mu}}(1+\frac{S}{2\pi R}) \end{aligned} \tag{22}$$

In the last step equation (21) has been used to substitute for ε. Equation (22) is interesting since it reveals that under the posed conditions, the total rendezvous time depends only on S. Moreover, it depends very weakly on S. The ratio S to 2πR is the ratio between the initial separation and the circumference of one target orbit!

Calculating the delta-V requirement is another exercise in careful bookkeeping. There will be a total of three impulsive burns to track. The first burn is the H-transfer from the original orbit to the intermediate inner orbit. The second burn is the immediate transfer from the inner orbit to the target orbit. The final burn will match the position and speed of the assembly point, thus completing the rendezvous maneuver. The results:

$$\begin{aligned} \Delta v = &\sqrt{\frac{\mu}{R+\delta}} - \sqrt{\mu(\frac{2}{R+\delta} - \frac{1}{R+\delta/2+\varepsilon/2})} \\ &+ \sqrt{\mu(\frac{2}{R+\varepsilon} - \frac{1}{R+\delta/2+\varepsilon/2})} \\ &- \sqrt{\mu(\frac{2}{R+\varepsilon} - \frac{1}{R+\varepsilon/2})} \\ &+ \sqrt{\frac{\mu}{R}} - \sqrt{\mu(\frac{2}{R} - \frac{1}{R+\varepsilon/2})} \end{aligned}$$

When linearized, the above expression reduces to:

$$
\Delta v = \sqrt{\frac{\mu}{R^3}} \left(\frac{\delta - \varepsilon}{2}\right) = \sqrt{\frac{\mu}{R^3}} \left(\frac{3}{4}\delta - \frac{S}{3\pi}\right) \tag{23}
$$

Again, equation (21) has been employed to substitute for ε. Note that when S = 3πδ/4 (corresponding to the critical angle α), that the solution for region II is retrieved. Note also the interesting reversal that has taken place. In regions II and IV, delta-V depended only on the orbital difference δ, while the rendezvous time was a function of both δ and S. Now, for region I, it is clear that rendezvous time depends only on S, while delta-V relies instead on both parameters.

In an effort to avoid needless repetition, it is true that the equations for region III match those of region I. The same symmetry exists between these two regions as it does for regions II and IV. It is assumed, of course, that appropriate boundary limits are applied, namely δ < 0 and S > 3πδ/4. In this case it is apparent that the space vehicle leads the assembly point and is moving away from it, since the vehicle is in a lower orbit. The desired action will be to transfer to an orbit above the final target orbit.

These equations were developed in an effort to analyze how fuel and time resources would be used for the given assembly algorithm. These equations apply to single vehicles, but make it possible to tackle some issues of the formation assembly problem more easily.

Results

To help analyze the results, a MATLAB script was created to do some calculations and graph the results. The results are shown in Figures 5, 6, and 7. For these results, a 620 km altitude orbit was examined (R = 7000 km). The initial separation distance was chosen to be S = -1000 m. Note that S is a free parameter, and that the results allow the behavior of the performance metrics to be examined for various values of δ. It is also possible to examine this behavior for a particular value of δ and various values of S. The following brief remarks will illustrate how the derived equations can be used to evaluate assembly operations.

Figure 5 shows a plot of the total transfer time versus δ for the described initial condition. Some of the characteristics of the assembly strategy can easily be seen. For instance, when δ is large and negative, the starting satellite position will be in region III. As seen from the analysis section, the transfer time in regions I and III is more or less constant. As δ increases, this implies that the starting position moves into region IV. Now a different set of transfer rules will dictate the spacecraft's behavior. As δ nears zero, the coast time becomes increasingly long (recall that the difference in angular rates becomes very small). Then, when δ = 0, the vehicle is in region I and must catch up according to the other set of assembly rules. Figure 5 also gives the first clue that the chosen algorithm is not an optimal one, because there is a discontinuity in the graph. Surely a small change in δ would not suddenly change the value of rendezvous time by thousands of seconds. There must be an orbital maneuver that provides a more gradual change. The algorithm that was analyzed was chosen partly due to mathematical convenience, and partly to illustrate this very point – that there is a need to be aware of the performance gaps in the operations scheme. For instance, it might be prudent to append some other rendezvous scheme for values of δ between –150 and 0 meters, which would help smooth out the curve and cover up the poor performance demonstrated by the current strategy.

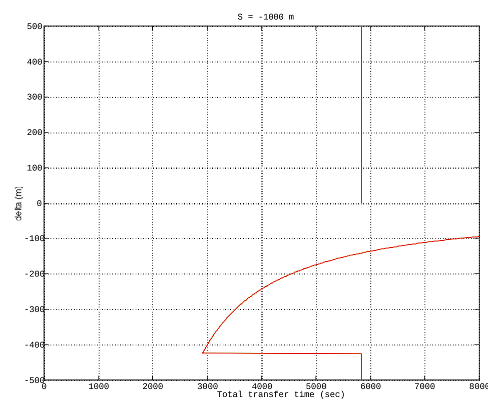

Figure 5 – Transfer time as a function of delta for the initial separation S = -1000 m

Figure 6 is a plot of the delta-V requirement as a function of δ. It should be noted that there may be more than one value of δ which corresponds to a particular value of delta-V. This seems reasonable, and may be expected by symmetry arguments. However, a noticeable discontinuity exists on this chart as well, when δ>0. This is another indicator that there is a performance gap in the chosen strategy: all of a sudden, as δ climbs into positive territory, the fuel usage jumps.

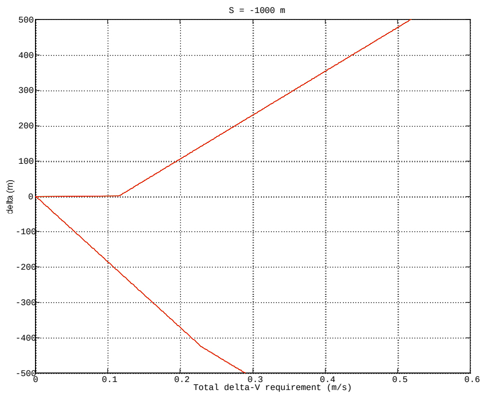

Figure 6 – Delta-V requirement as a function of delta for the initial separation S = -1000 m

The most telling graph is Figure 7. This chart plots the total transfer time against the corresponding required delta-V values. This is a very useful tool. Form this chart and for the given initial condition, a desired transfer time-fuel consumption pair of values can be selected. Back-reference to the previous charts then will dictate what value of δ should be used. This last point is an important one: the ultimate goal of planning an orbit assembly (either by ground personnel or autonomously by the space formation) will be in choosing an assembly point. Charts like Figure 7 allow the impact on mission resources to be compared for the various choices.

As an example, consider the assembly strategy represented in Figure 7, which offers two choices of delta-V for each choice of transfer time. It is much more desirable to choose a value of δ which corresponds to the curvy part of the chart, since that region will require less fuel over a shorter time period (incidentally, that portion of the graph represents a starting position in region IV). However, a second satellite in this group, using the same assembly strategy, may not be so lucky. By choosing that value of δ for the first satellite, the second satellite may find that its time-delta-V data point resides in a performance-poor position on its own chart. This is the operational hazard and the tradeoff must be performed using tools such as these.

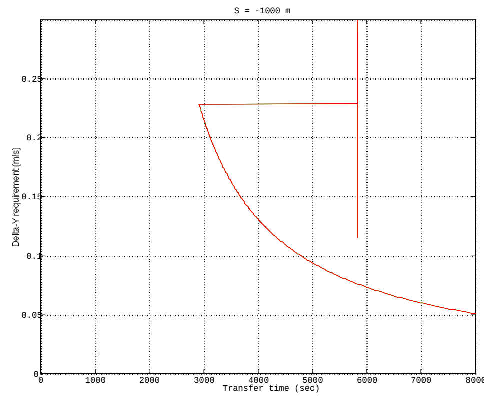

Figure 7 – Delta-V – Transfer time relationship for S = -1000 m

Recommendations and Conclusions

It is not truthful to state that this solution is better than any other solution. Some of the assumptions made in the presented model could easily be refuted when considering other missions. However, the method of thinking applied here may be extended to many other formation assembly strategies. The emphasis on identifying precious resources, and being able to analyze how proposed operations concepts will affect them is the key.

It is worth noting that a great deal of future work can be derived from this study. In order to foster brevity, only one formation assembly method was solved. The discussion was intended to focus on how to derive relationships between precious resources in order to get an idea about operational constraints. No attempt was made to truly trade-off or "optimize" resources in an actual formation assembly scenario. This would be an excellent avenue to explore. Also, using the derived equations and the given method to manage a large number of satellites (more than two) could reveal some useful operational insights that were not reported here. Furthermore, extending the reach of this approach to include non-circular orbits or three-dimensional maneuvering could also be a valuable contribution.

In summary, the increased interest in using large numbers of small satellites for space missions has garnered a great deal of attention. As such, one class of missions under this category, formation flying, has been singled out for study due to its many potential performance benefits. While a great deal of effort has been expended studying methods of controlling a formation of spacecraft, it was noted that little effort had been spent understanding how the vehicles were to rendezvous in the first place. This process was named formation assembly. It was then shown that for a reasonable assembly strategy, useful equations could be derived that described the consumption of critical system resources. The main resources looked at were time and fuel. Finally, it was concluded that studying these equations could indicate what assembly-transfer regions were more beneficial than others. This information is useful when attempting to choose a location to assemble the formation.

Bibliography

1. Conway, B. and Prussing, J.E. Orbital Mechanics. Oxford University Press, New York, 1993.

2. *Orion Mission Requirements Document*. Orion design team. 1998

3. Larson, W. and Wertz, J., editors. Space Mission Analysis and Design 2nd Edition. 1991